

Bayerisches Zentrum für
Angewandte Energieforschung e.V.

Thermal Conductivity of PCM

Introduction

Hans-Peter Ebert

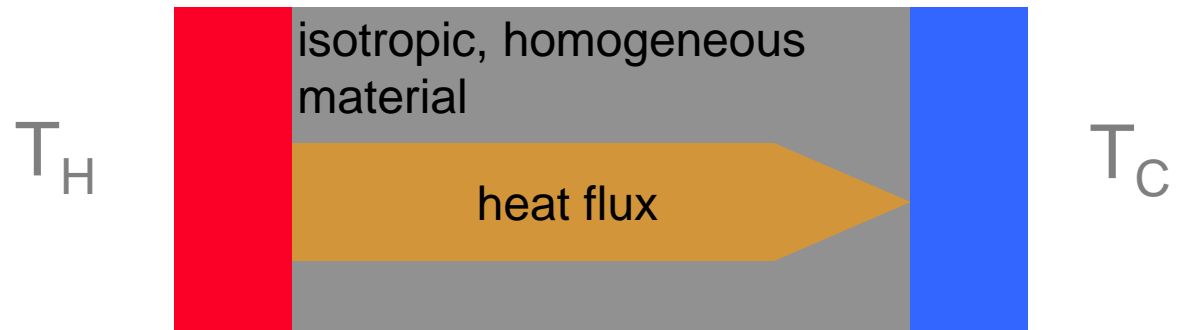
OVERVIEW

- Motivitation
- Thermal conductivity · thermal diffusivity
- Measurement methods
- Final remarks



Fourier's law defines the thermal conductivity λ :

$$\vec{q} = -\lambda \nabla T$$



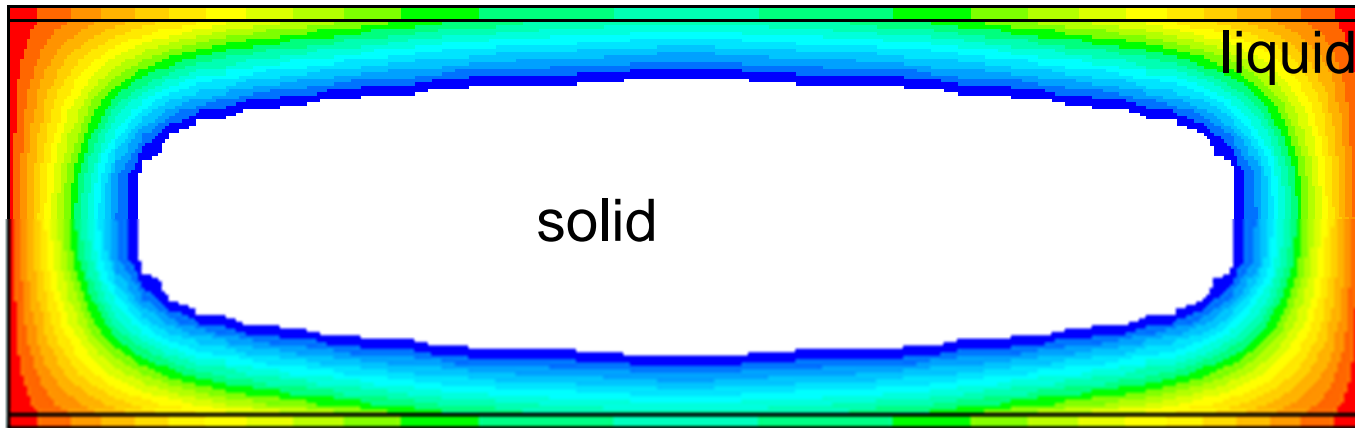
Jean Baptiste Joseph Fourier
(1768 - 1830)

Thermal conductivity λ is a material property!

$$[\lambda] = \frac{W}{m^2} \cdot \frac{m}{K} = \frac{W}{m \cdot K}$$

WHY THERMAL CONDUCTIVITY HAS TO BE KNOWN?

- input data for simulations



- product comparison (product data sheets)

In all cases you need reliable thermal conductivity data!

EQUATION OF HEAT TRANSFER

Energy balance of a volume V with surface S
between the times t_1 and t_2



homogeneous
medium

EQUATION OF HEAT TRANSFER

$$\int_{t_1}^{t_2} \int_S \lambda(\vec{x}) \nabla T d\vec{f} dt + \int_{t_1}^{t_2} \int_V \Phi(\vec{x}, t) d^3 \vec{x} dt = \int_V c(\vec{x}) \rho(\vec{x}) [T(\vec{x}, t_2) - T(\vec{x}, t_1)] d^3 \vec{x}$$

net heat flow via the
surface S

heat source or
sink

increase of the internal energy due to a
increase of temperature

Φ = heat source (z.B. phase change, chemical reaction, radiative heat transfer)

ρ = density

c = specific heat

λ = thermal conductivity

T = temperature

EQUATION OF HEAT TRANSFER

$$\int_{t_1}^{t_2} \int_S \lambda(\vec{x}) \nabla T d\vec{f} dt + \int_{t_1}^{t_2} \int_V \Phi(\vec{x}, t) d^3 \vec{x} dt = \int_V c(\vec{x}) \rho(\vec{x}) [T(\vec{x}, t_2) - T(\vec{x}, t_1)] d^3 \vec{x}$$

$$\int_S \lambda(\vec{x}) \nabla T d\vec{f} = \int_V \nabla(\lambda(\vec{x}) \nabla T) d^3 \vec{x}$$

$$[T(\vec{x}, t_2) - T(\vec{x}, t_1)] = \int_{t_1}^{t_2} \frac{\partial T}{\partial t} dt$$

$$\int_{t_1}^{t_2} \int_V \left\{ \nabla(\lambda(\vec{x}) \nabla T(\vec{x}, t)) + \phi(\vec{x}, t) = \rho(\vec{x}) c(\vec{x}) \frac{\partial T(\vec{x}, t)}{\partial t} \right\} d^3 \vec{x} dt$$

General Equation of Heat Transfer
(Parabolic partial differential equation)

$$\nabla(\lambda(\vec{x}) \nabla T(\vec{x}, t)) + \phi(\vec{x}, t) = \rho(\vec{x}) c(\vec{x}) \frac{\partial T(\vec{x}, t)}{\partial t}$$

SIMPLIFIED ONE-DIMENSIONAL EQUATION OF HEAT TRANSFER

$$\frac{\partial}{\partial x} \left(\lambda(x) \frac{\partial}{\partial x} T(x, t) \right) + \phi(x, t) = \rho(x) c(x) \frac{\partial T(x, t)}{\partial t}$$

- no sources of heat
- constant thermal conductivity, density and specific heat

$$\frac{\partial^2}{\partial x^2} T(x, t) = \frac{\rho \cdot c}{\lambda} \frac{\partial T(x, t)}{\partial t}$$

$$\frac{\partial^2}{\partial x^2} T(x, t) = \frac{1}{\alpha} \frac{\partial T(x, t)}{\partial t}$$

α : thermal diffusivity

SIMPLIFIED ONE-DIMENSIONAL EQUATION OF HEAT TRANSFER

$$\frac{\partial^2}{\partial x^2} T(x, t) = \frac{\rho \cdot c}{\lambda} \frac{\partial T(x, t)}{\partial t}$$

Stationary heat transfer:

$$\frac{\partial^2}{\partial x^2} T(x) = 0$$

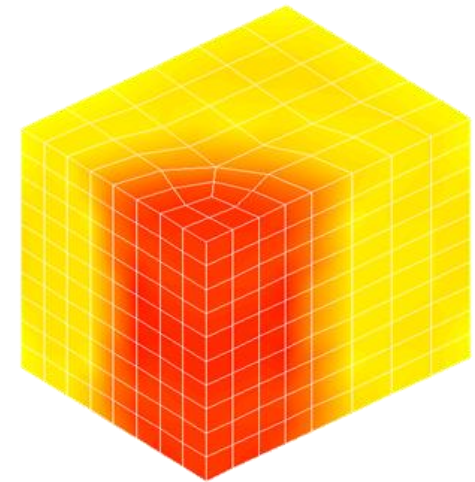
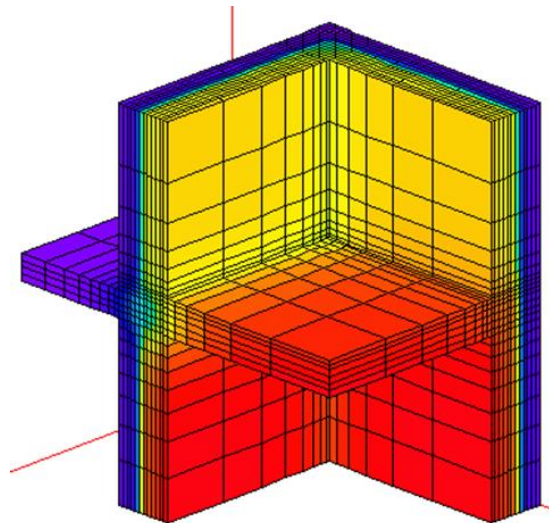
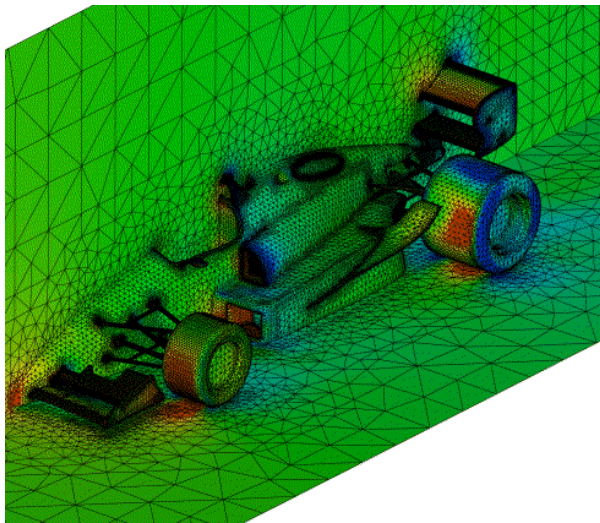
$$\frac{\partial}{\partial x} T(x) = \text{const.} = -\frac{q}{\lambda}$$

PRINCIPLES OF THERMAL SIMULATIONS

Common methods: finite-difference-, finite-volume-method, analytical methods

$$\rho \cdot c_p \cdot \frac{\partial T}{\partial t} = \lambda \cdot \frac{\partial^2 T}{\partial x^2} \quad \Rightarrow \quad \rho \cdot c_p \cdot \frac{T_i^{n+1} - T_i^n}{\Delta t} = \lambda \cdot \frac{T_{i+1}^n - 2 \cdot T_i^n + T_{i-1}^n}{(\Delta x)^2}$$

In all cases **Equation of Heat Transfer** is the underlying “working equation” and conduction effects are separated from heat storage effects.



HOW WE COULD DETERMINE THERMAL CONDUCTIVITY?

Fourier's law

$$\vec{q} = -\lambda \nabla T$$

Prerequisites:

- stationary conditions
- determination of the temperature gradient
- determination of the heat flow

Equation of heat transfer

$$\lambda \Delta T(\vec{x}, t) = \rho \cdot c \frac{\partial T(\vec{x}, t)}{\partial t}$$

Prerequisites:

- instationary conditions
- defined boundary and initial conditions
- known solution of the equation of heat transfer

COMPARISON OF STATIONARY AND NON-STATIONARY METHODS

NON-STATIONARY methods

STATIONARY methods

Solution of heat transfer equation is needed

Simple theory

„Short“ measurement times

Stationary conditions are needed

Inhomogeneous specimens can be critical

Integration over the measurement area – larger specimens can be investigated

Small specimen dimensions can be investigated,
Advantageous for high T and high p

Large specimens

Thermal contact resistances not always critical

Thermal contact resistances are critical

- It is no problem if the experiment is conducted far below the melting point > determination of λ_s .
- It is no problem if the experiment is conducted far above the melting point > determination of λ_l .
- The thermal conductivity of the liquid phase is lower than the thermal conductivity of the solid phase.
- In the case that two phases exist within the specimen during a thermal conductivity measurement the results have to be carefully discussed.

ROUND ROBIN TEST



ZAE BAYERN
Bayerisches Zentrum
für Angewandte
Energieforschung

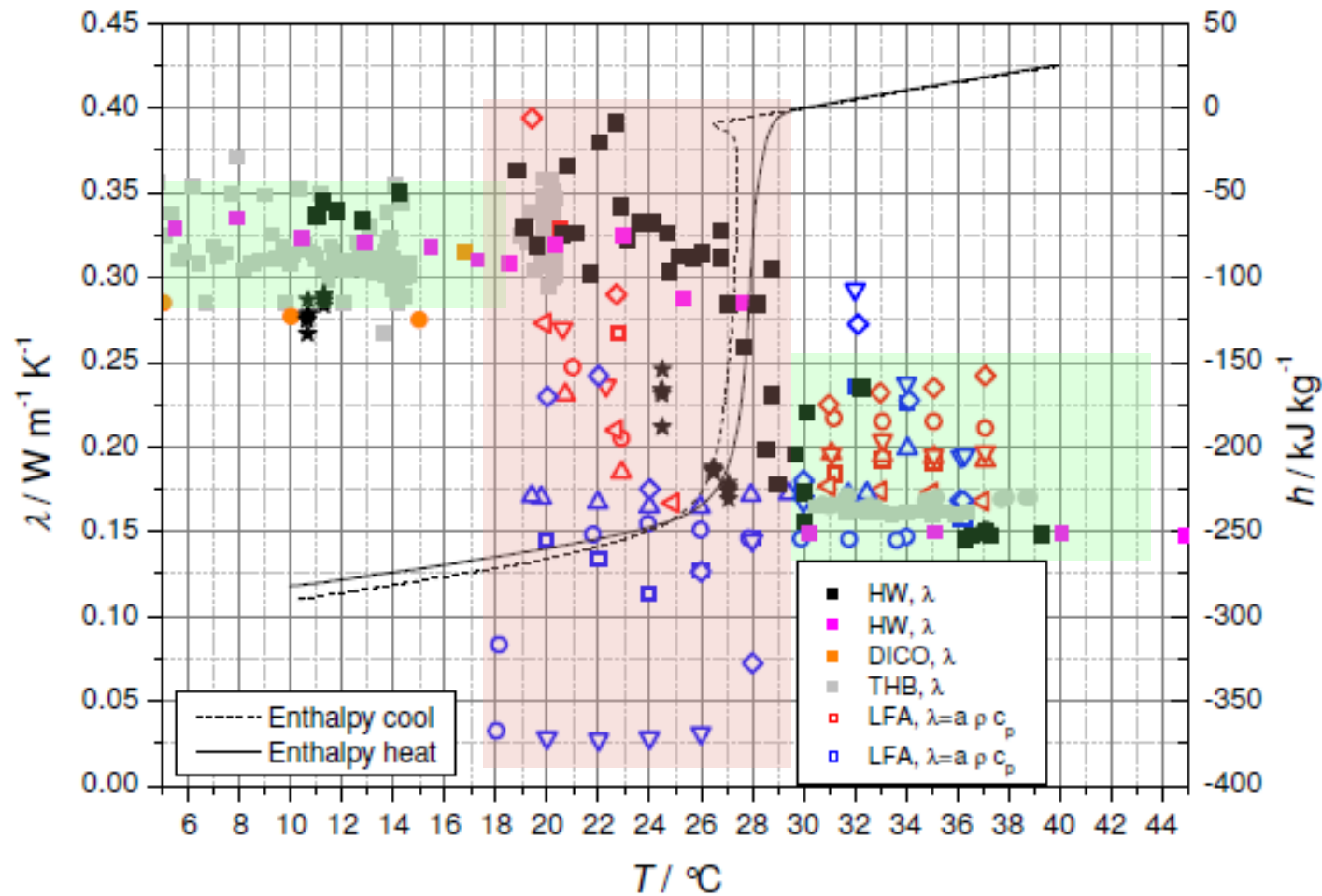
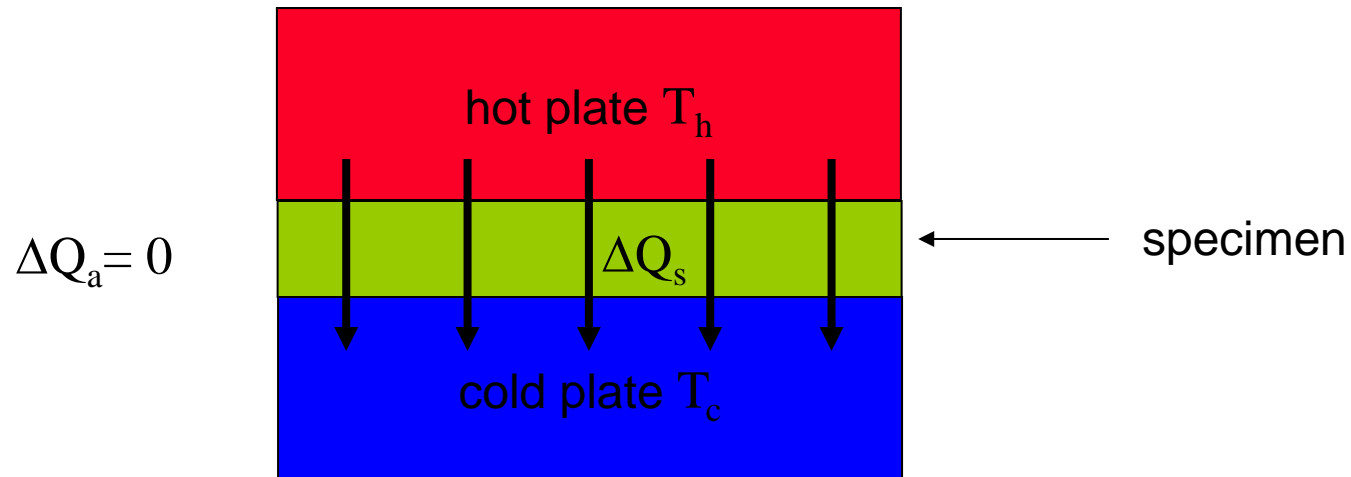


Figure 1: Thermal conductivity of n-octadecane measured with different measurement methods

STATIONARY GUARDED HOT PLATE METHOD



$$\Delta Q = \Lambda \cdot A \cdot (T_h - T_c) \cdot \Delta t$$

with: measurement area A , thermal conductivity λ und thermal conductance Λ

$$\Lambda = \frac{\lambda}{d}$$

with: d = specimen thickness

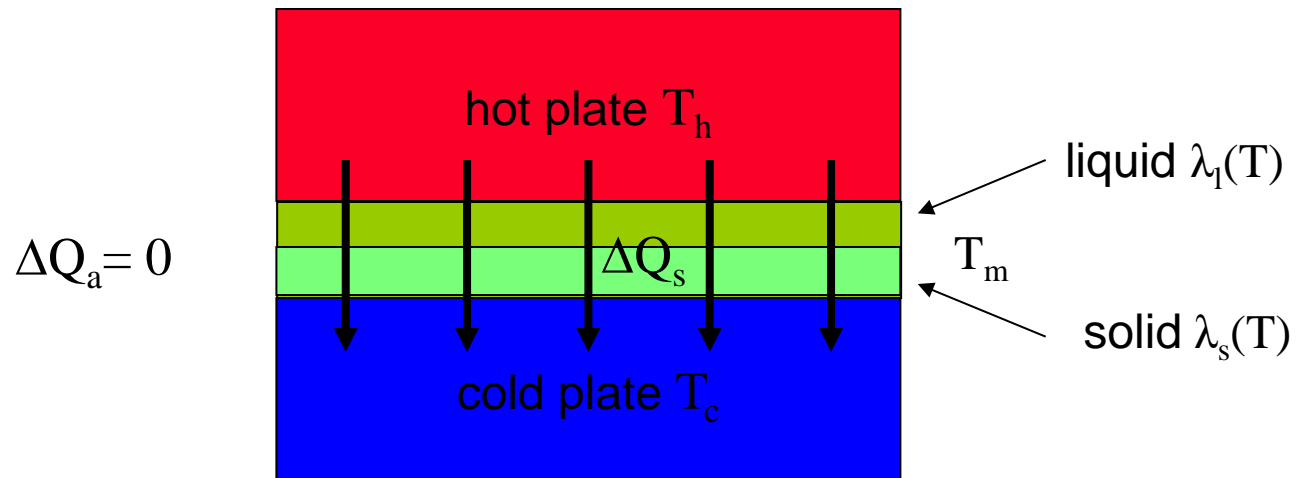
STATIONARY GUARDED HOT PLATE METHOD

$$T_h > T_m > T_c$$



ZAE BAYERN

Bayerisches Zentrum
für Angewandte
Energieforschung

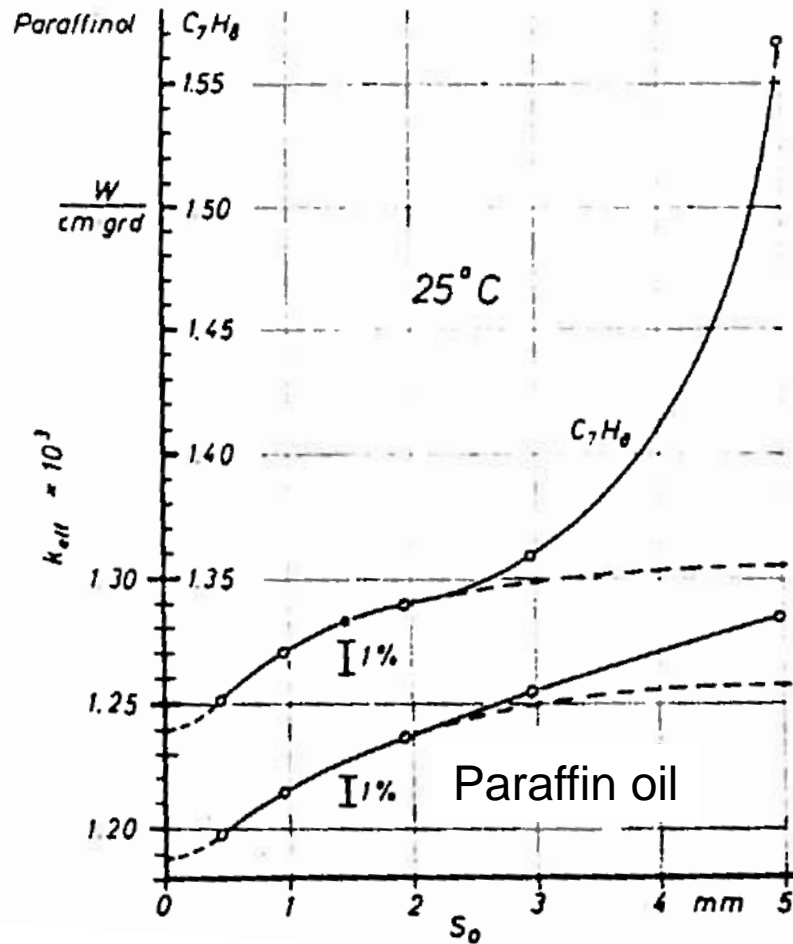


STATIONARY GUARDED HOT PLATE METHOD

Some remarks about radiative transfer within paraffins



ZAE BAYERN
Bayerisches Zentrum
für Angewandte
Energieforschung



← convection effect
← radiation effect (+ 7% !)

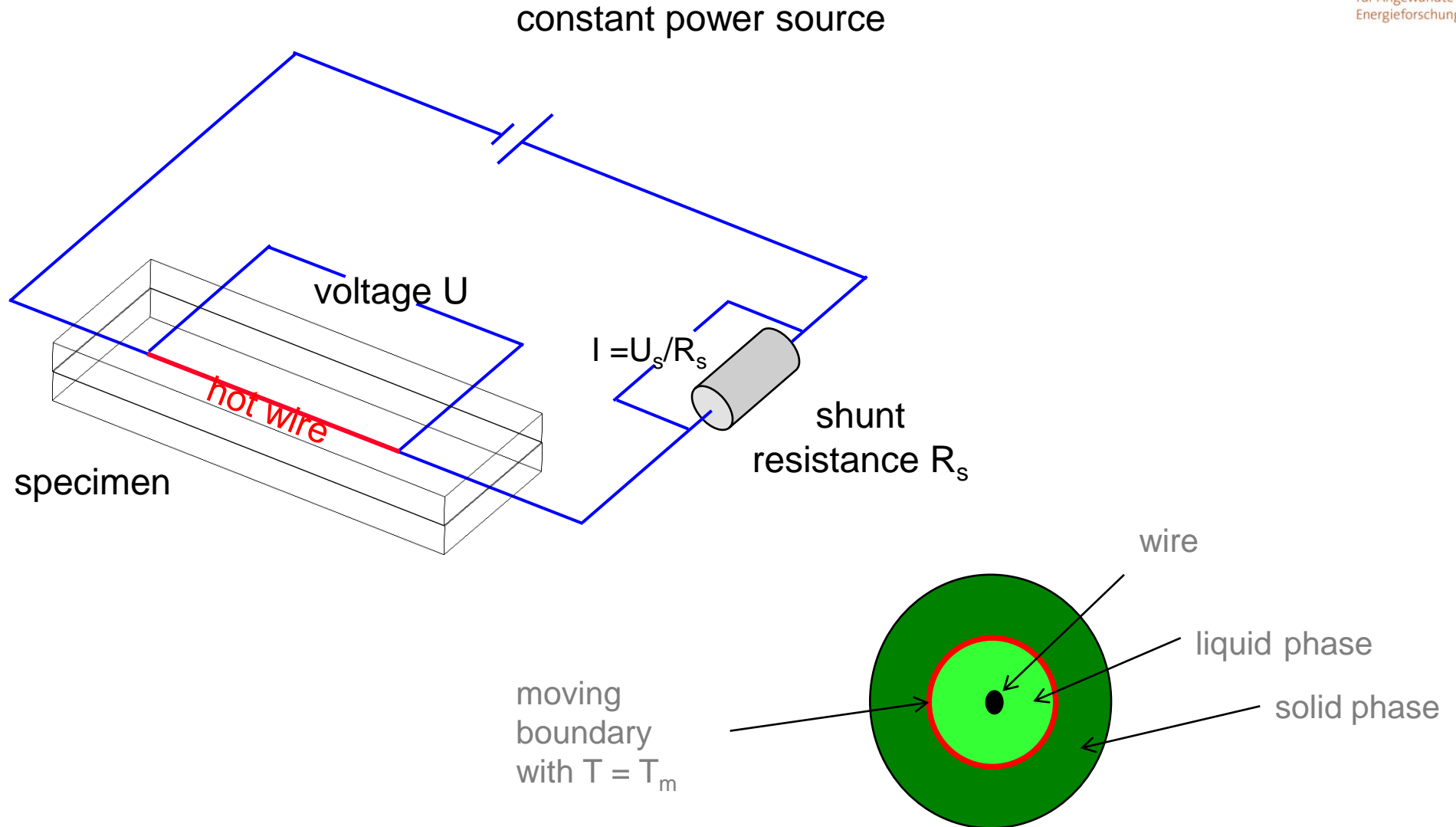
Poltz, int. J. Heat Mass Transfer Vol. 8 pp. 609 – 620 (1964)
Thermal Conductivity of Liquids III

DYNAMIC HOT-WIRE METHOD

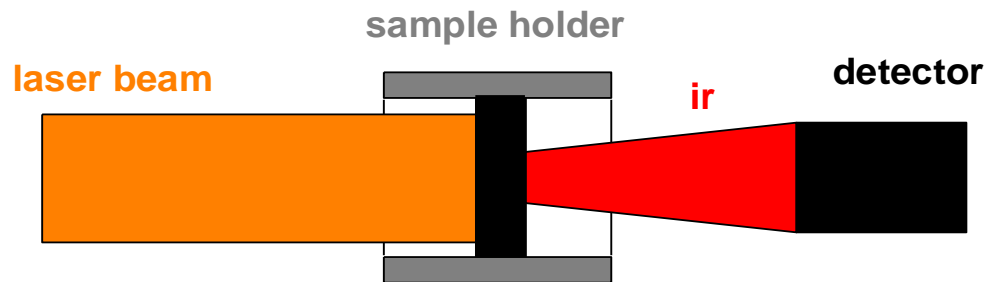


ZAE BAYERN

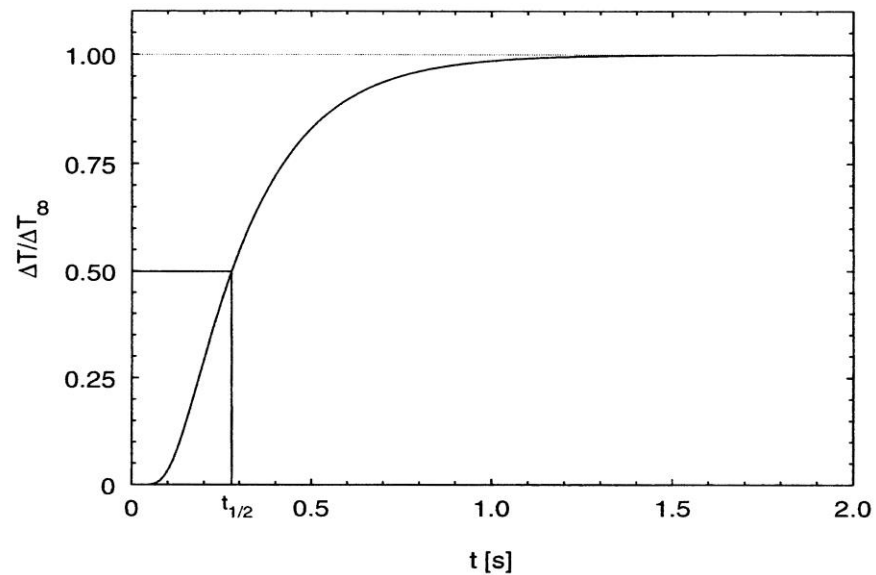
Bayerisches Zentrum
für Angewandte
Energieforschung



$$\lambda(T) = a(T) \cdot \rho(T) \cdot c_p(T)$$



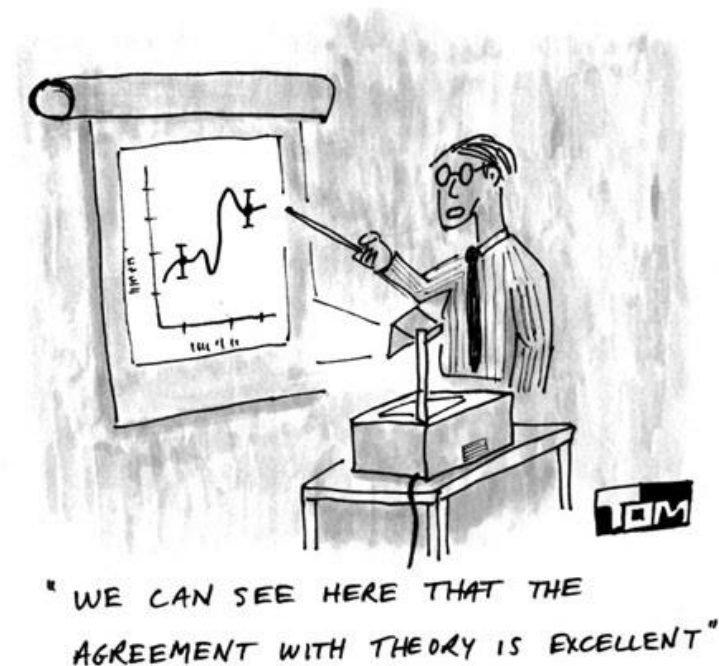
$$a = 1.38 \cdot \frac{l^2}{\pi^2 t_{1/2}}$$



What is the result of an evaluation procedure if the underlying working equation meets not the experimental reality?

Facts:

- The working equation considers not the phase transition (in the most cases).
- Fitting algorithms will deliver a best fit solution – in respect to the laws of mathematic – not in respect to the physics.



©2000 Tom Swanson